Matrix Factorization and Collaborative Filtering

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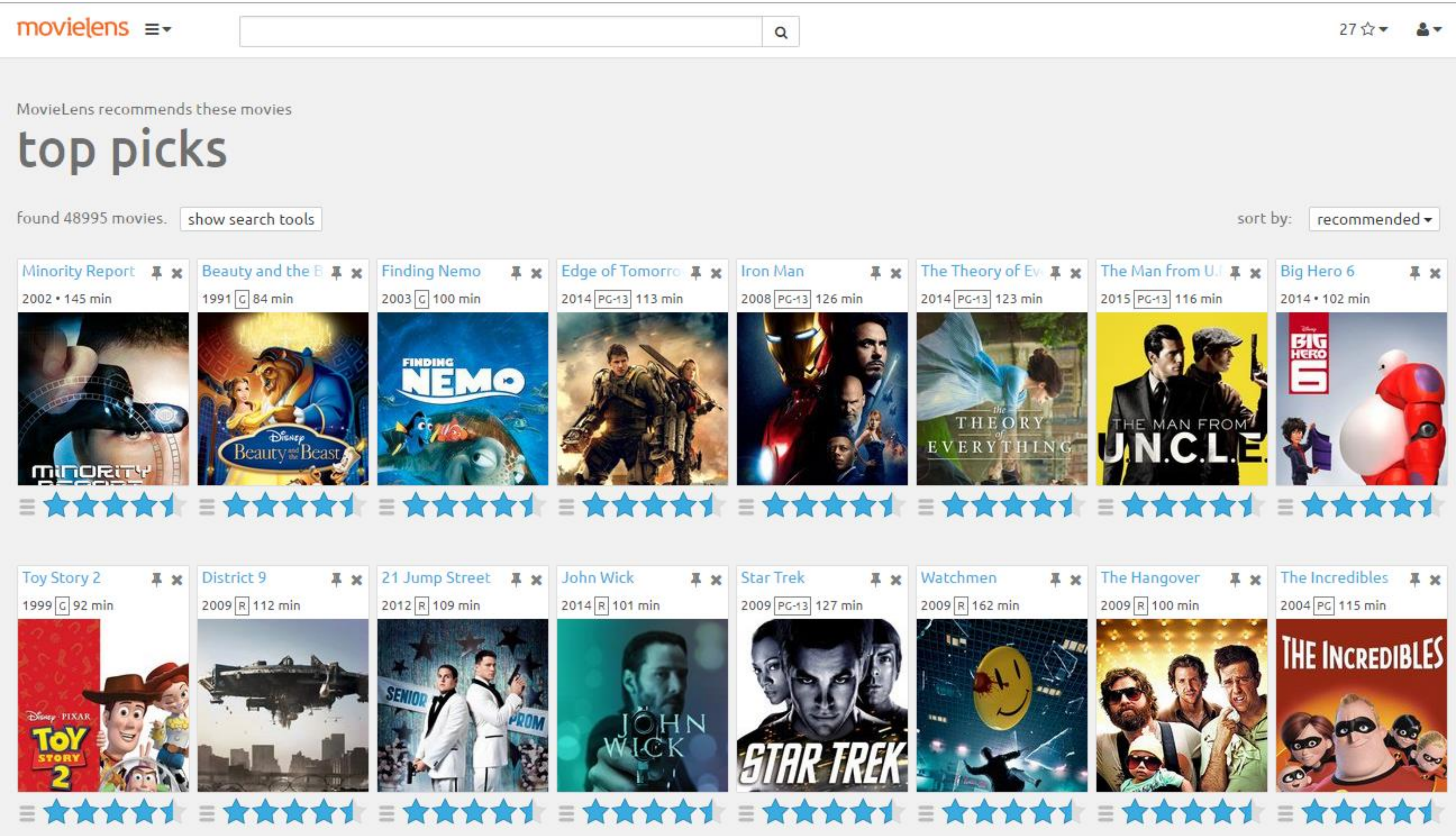
Outline

- **Recommender System**
- Matrix Factorization
- Reference

Recommender System



Recommender System Strategies

1. Content Filtering
   - Create a profile for each user or product to characterize its nature

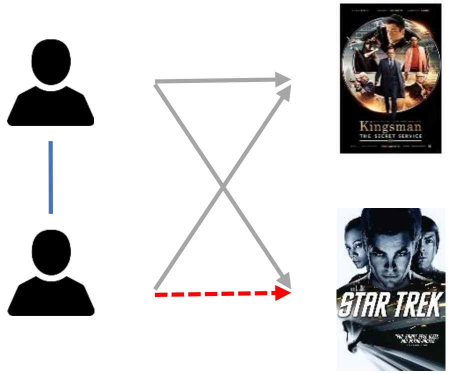
Genres: Crime, Comedy, Action, Adventure
Directors: Matthew Vaughn
Cast: Taron Egerton, Colin Firth, Samuel L. Jackson, …
Distributor: Fox
Box Office Popularity: …

Gender
Region
…
Answers provided on a suitable questionnaire

Recommender System Strategies

2. Collaborative filtering
   - Rely only on past user behavior
   - Everyday examples
     - Bestseller lists
     - Top 40 music lists
     - Unmarked but well-used paths thru the woods
     - The "recent returns" shelf at the library

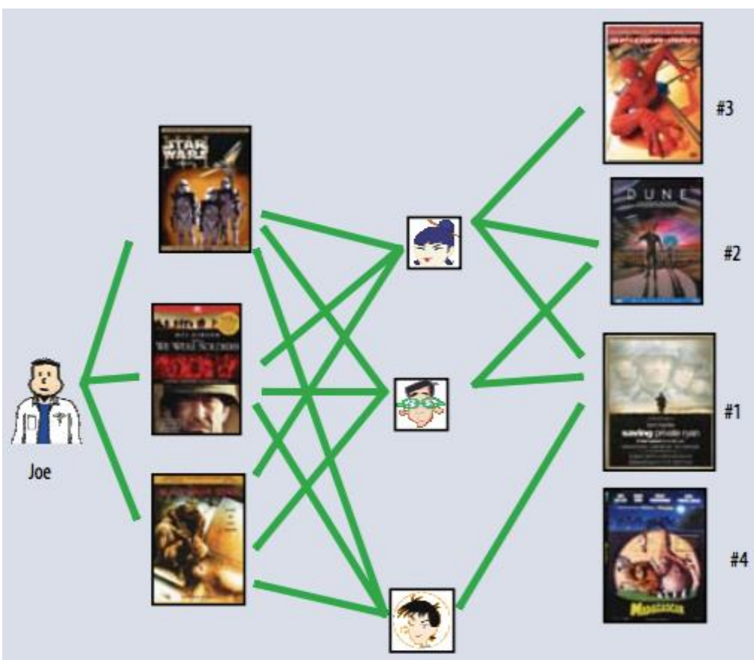
   - Common insight: **personal tastes are correlated**

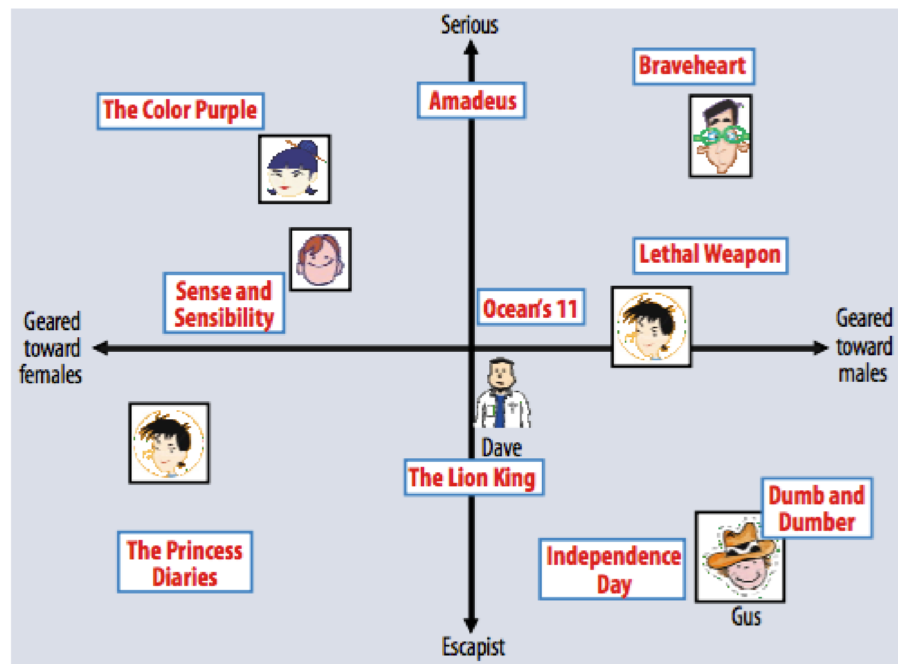


Types of Collaborative Filtering

a. Neighborhood Methods
   - Find neighbors based on similarity of movie preferences
   - Recommend movies that those neighbors watched

b. Latent Factor Methods
   - Characterize both items and users
   - Recommend a movie based on its proximity to the user in the latent space





* Figures in [3]

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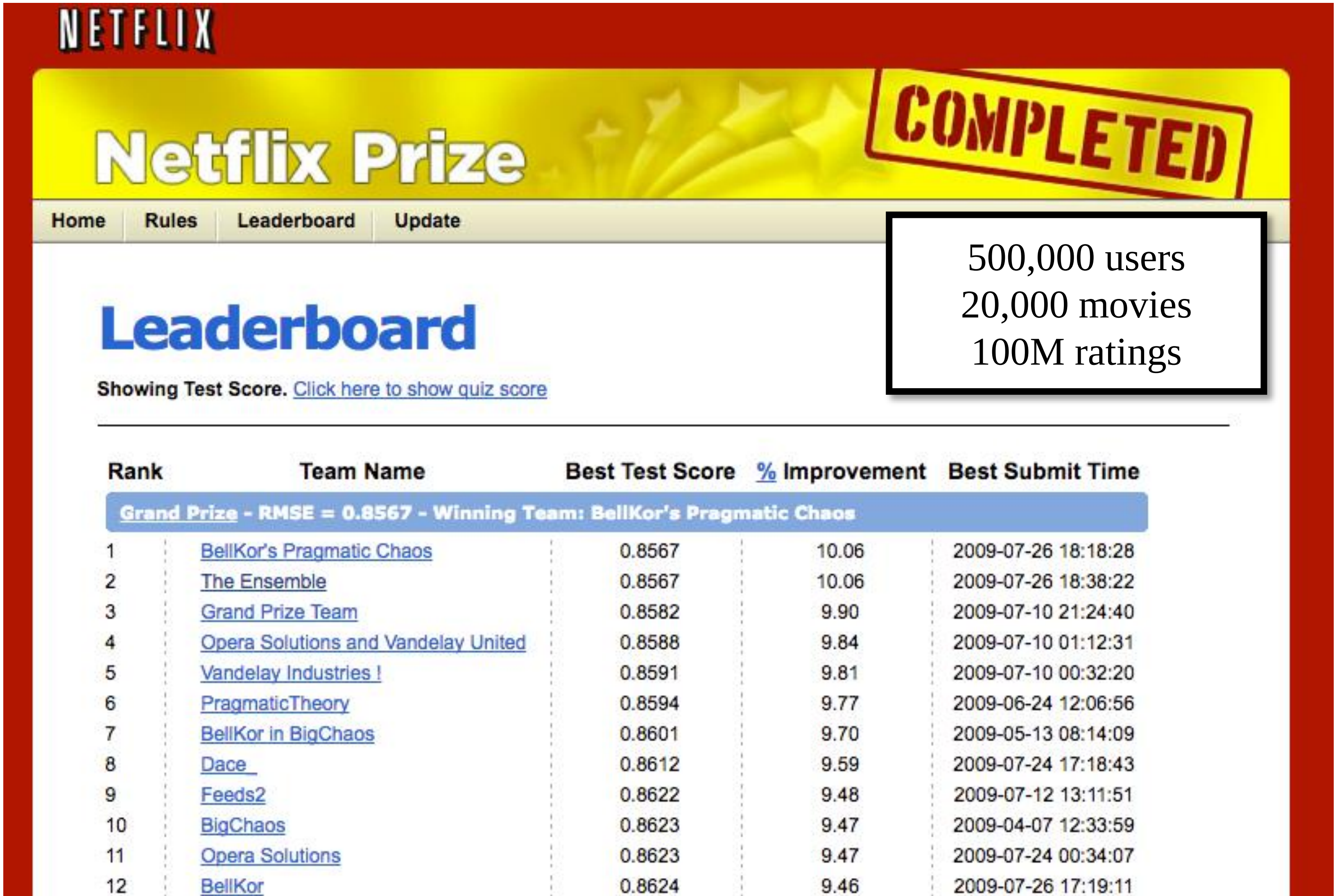
Netflix Prize


NETFLIX

Netflix Prize

COMPLETED

Home | Rules | Leaderboard | Update

Leaderboard

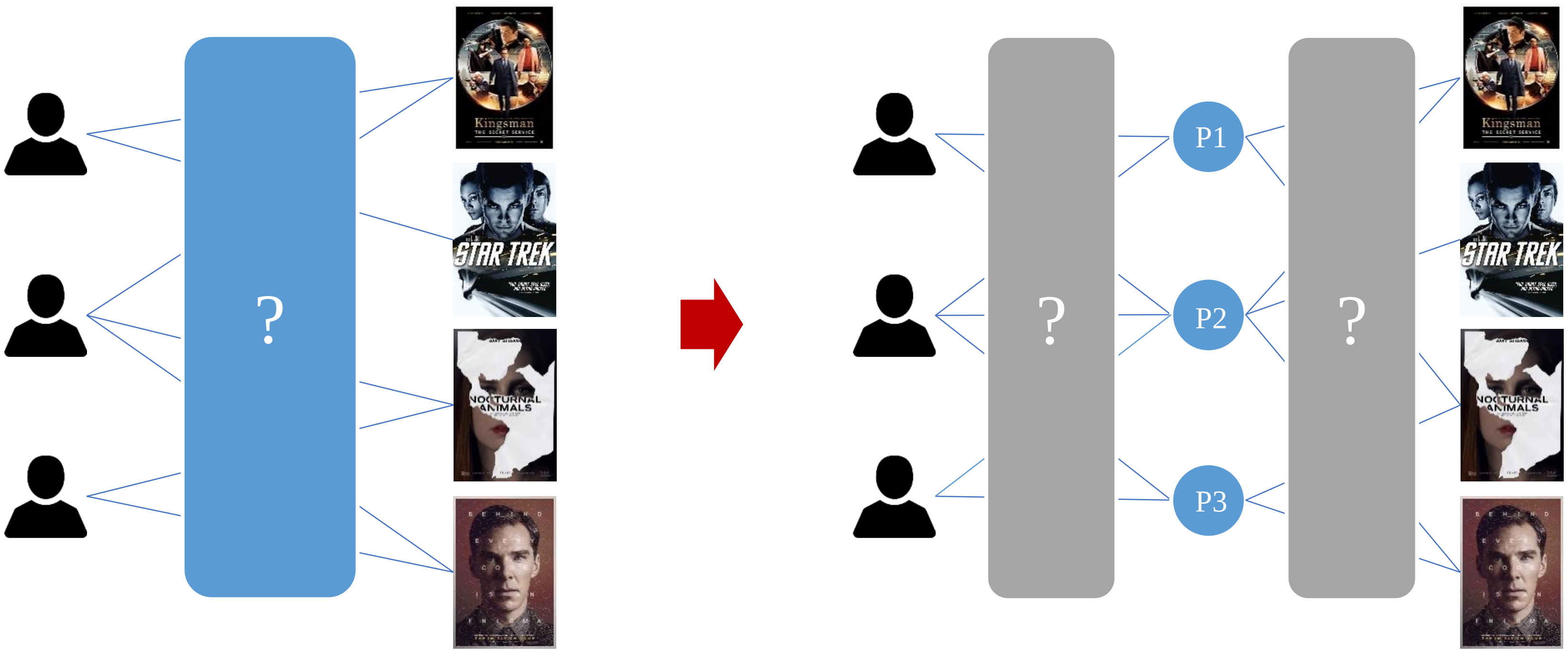
Showing Test Score. Click here to show quiz score

Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
Grand Prize - RMSE = 0.8567 - Winning Team: BellKor's Pragmatic Chaos				
1	BellKor's Pragmatic Chaos	0.8567	10.06	2009-07-26 18:18:28
2	The Ensemble	0.8567	10.06	2009-07-26 18:38:22
3	Grand Prize Team	0.8582	9.90	2009-07-10 21:24:40
4	Opera Solutions and Vandelay United	0.8588	9.84	2009-07-10 01:12:31
5	Vandelay Industries !	0.8591	9.81	2009-07-10 00:32:20
6	PragmaticTheory	0.8594	9.77	2009-06-24 12:06:56
7	BellKor in BigChaos	0.8601	9.70	2009-05-13 08:14:09
8	Dace_	0.8612	9.59	2009-07-24 17:18:43
9	Feeds2	0.8622	9.48	2009-07-12 13:11:51
10	BigChaos	0.8623	9.47	2009-04-07 12:33:59
11	Opera Solutions	0.8623	9.47	2009-07-24 00:34:07
12	BellKor	0.8624	9.46	2009-07-26 17:19:11

500,000 users
20,000 movies
100M ratings

Matrix Factorization

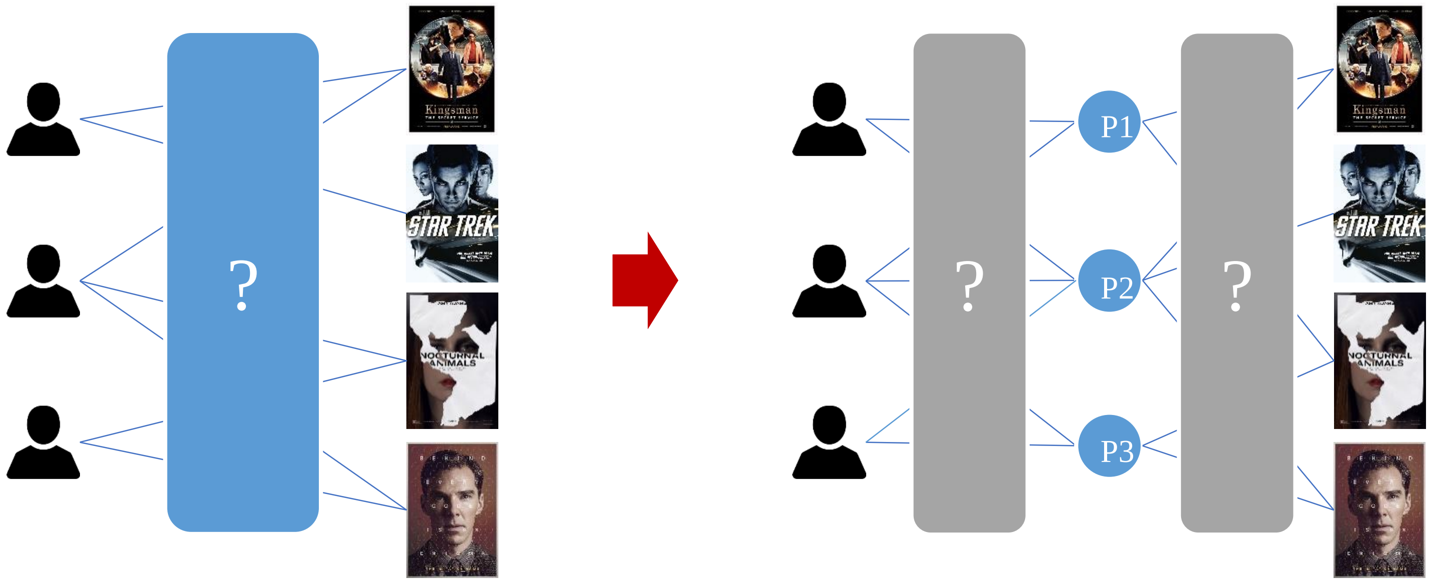
- Assume latent factors in user preference



* Redraw figures in [2]

Matrix Factorization

- Assume latent factors in user preference



	I_1	I_2	I_3	I_4
U_1				
U_2				
U_3				

	P_1	P_2	P_3
U_1			
U_2			
U_3			

	I_1	I_2	I_3	I_4
P_1				
P_2				
P_3				

* Redraw figures in [2]

Singular Value Decomposition

$$A = U \times s \times V^T$$

```
[ 4.  2.  3.  5.  1.]
[ 0.  3.  0.  4.  2.]
[ 5.  4.  3.  3.  0.]
[ 0.  0.  5.  5.  2.]
[ 5.  0.  0.  5.  0.]
```

=

```
[-0.54   0.03  -0.021  0.099 -0.835]
[-0.29  -0.225  0.393 -0.84   0.07 ]
[-0.506  0.372  0.574  0.374  0.371]
[-0.417 -0.79  -0.217  0.277  0.279]
[-0.441  0.432 -0.685 -0.26   0.287]
```

×

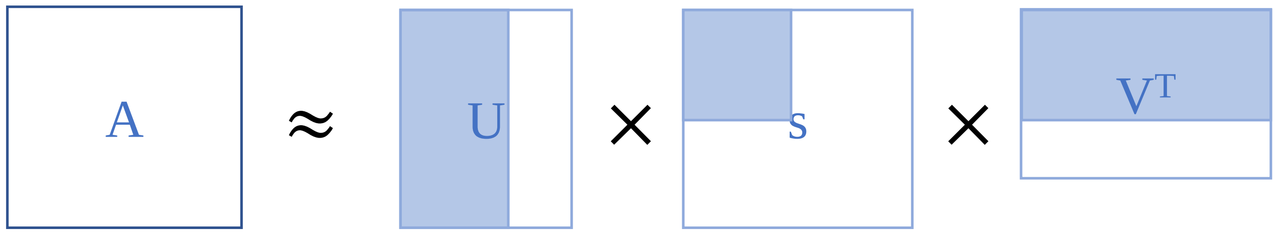
```
[ 13.707   0.      0.      0.      0.   ]
[  0.      5.607   0.      0.      0.   ]
[  0.      0.      3.791   0.      0.   ]
[  0.      0.      0.      3.645   0.   ]
[  0.      0.      0.      0.      0.155]
```

×

```
[-0.503 -0.29  -0.381 -0.705 -0.143]
[ 0.738  0.156 -0.489 -0.254 -0.357]
[-0.169  0.905  0.15  -0.35   0.087]
[ 0.265 -0.227  0.769 -0.455 -0.282]
[-0.321  0.147  0.029  0.33  -0.875]
```

We can drop less important information

Singular Value Decomposition

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[-0.54    0.03   -0.021  0.099 -0.835]
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[  0.      0.      3.791   0.      0.   ]
[  0.      0.      0.      0.      0.   ]
[  0.      0.      0.      0.      0.   ]
```

×

```
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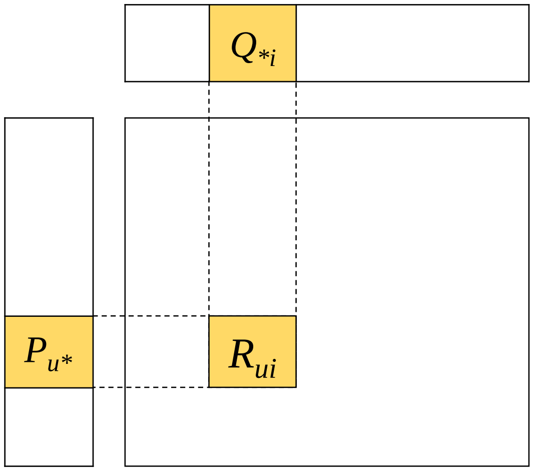
[-0.321  0.147   0.029  0.33   -0.875]
```

Matrix Factorization

- Matrices
  - User vector
    - $(P_{u*})^T \in \mathbb{R}^f$
  - Item vectors:
    - $(Q_{*i}) \in \mathbb{R}^f$
  - Rating prediction
    - $R_{ui} = P_{u*}Q_{*i} = [PQ]_{ui}$



- Vectors
  - User vector
    - $p_u \in \mathbb{R}^r$
  - Item vectors:
    - $q_i \in \mathbb{R}^r$
  - Rating prediction
    - $\hat{r}_{ui} = q_i^T p_u$
  - Set of non-zero entries
    - $\kappa = \{(u,i)\colon r_{ui} \neq 0\}$
  - Objective
    - $\min_{q*,p*} \sum_{(u,i)\in\kappa} (r_{ui} - q_i^T p_u)^2$

* Figures in [1]

Matrix Factorization

$$R_{ui} \approx \hat{R}_{ui} = U \times I$$

- Minimize the error between R and $\hat{\mathrm{R}}$

$$\min_{q*,p*} \sum_{(u,i)\in\kappa} (r_{ui} - q_i^T p_u)^2$$

$$\min_{q*,p*} \sum_{(u,i)\in\kappa} (r_{ui} - q_i^T p_u)^2 + \boxed{\lambda(\|q_i\|^2 + \|p_u\|^2)}$$

Regularization factor
- avoid overfitting
- make simple model

* Figures in [2]

Matrix Factorization

- How to deal with empty cells in matrix
  - With 0
  - With the average of the whole users
  - With the average of each user

	I_1	I_2	I_3	I_4
U_1		3	4	2
U_2	5			
U_3	3		2	

- Consider user bias and item bias

$$\boldsymbol{b_{ui} = \mu + b_i + b_u}$$

μ: average of the whole users

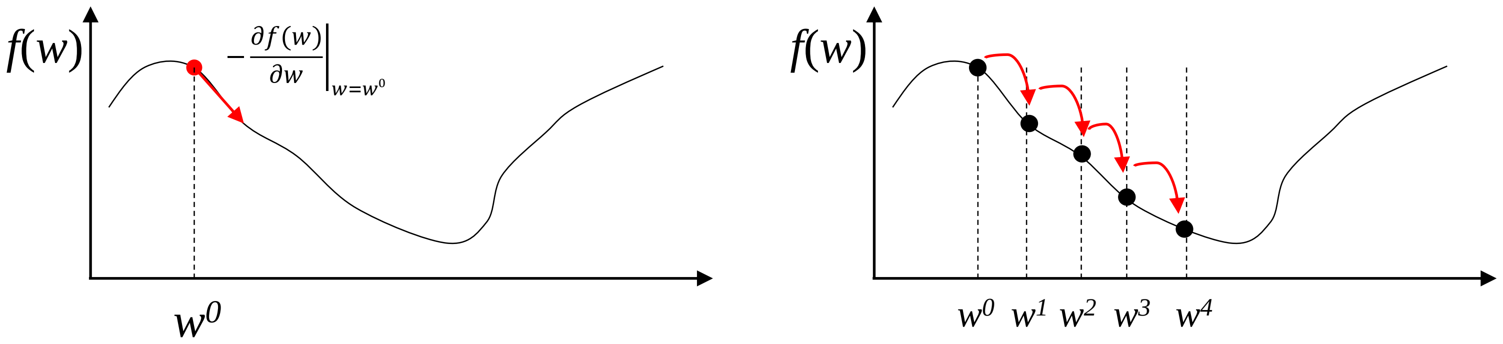
b_i, b_u: the observed deviations of u and i

$$\hat{r}_{ui} = \mu + b_i + b_u + q_i^T p_u$$

$$\min_{q*,p*,b*} \sum_{(u,i)\in\kappa} (r_{ui} - \mu - b_i - b_u - p_u^T q_i)^2 + \lambda(\|q_i\|^2 + \|p_u\|^2 + b_u^2 + b_i^2)$$

Matrix Factorization

- Approaches to minimizing $\min_{q*,p*} \sum_{(u,i)\in\kappa} (r_{ui} - q_i^T p_u)^2 + \lambda(\|q_i\|^2 + \|p_u\|^2)$
  1. Stochastic gradient descent



- Associated prediction error e_{ui}
  - $q_i \leftarrow q_i + \gamma(e_{ui} \cdot p_u - \lambda \cdot q_i)$
  - $p_u \leftarrow p_u + \gamma(e_{ui} \cdot q_i - \lambda \cdot p_u)$

* Figures in [2]

Matrix Factorization

- Approaches to minimizing $\min_{q*,p*} \sum_{(u,i)\in\kappa} (r_{ui} - q_i^T p_u)^2 + \lambda(\|q_i\|^2 + \|p_u\|^2)$
  2. Alternating least squares
     - Rotate between fixing the q_i's and fixing the p_u's
       - When all p_u's are fixed, the system recomputes the q_i's by solving a least-squares problems, and vice versa
     - Stochastic gradient descent is easier and faster than ALS in general, ALS is favorable in at least two cases
       - When the system can use parallelization
       - For systems centered on implicit data

Accuracy of Matrix Factorization Models

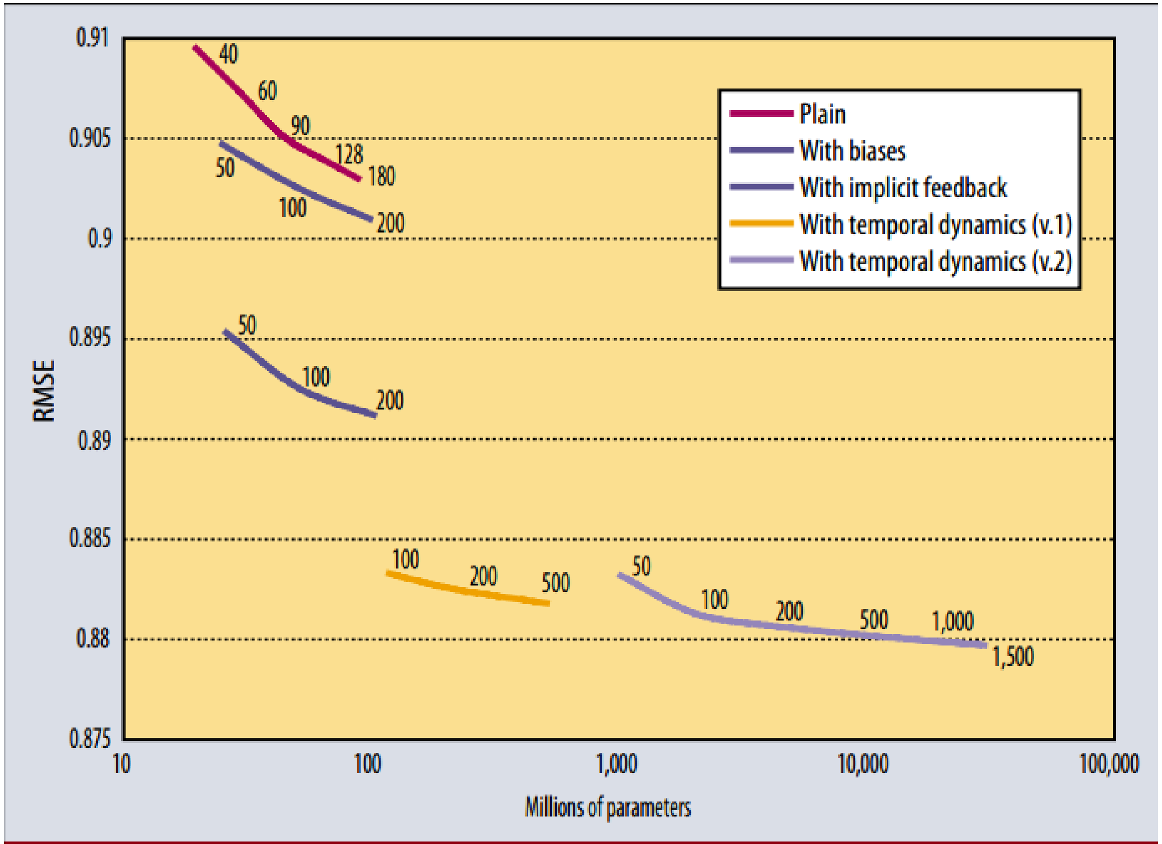
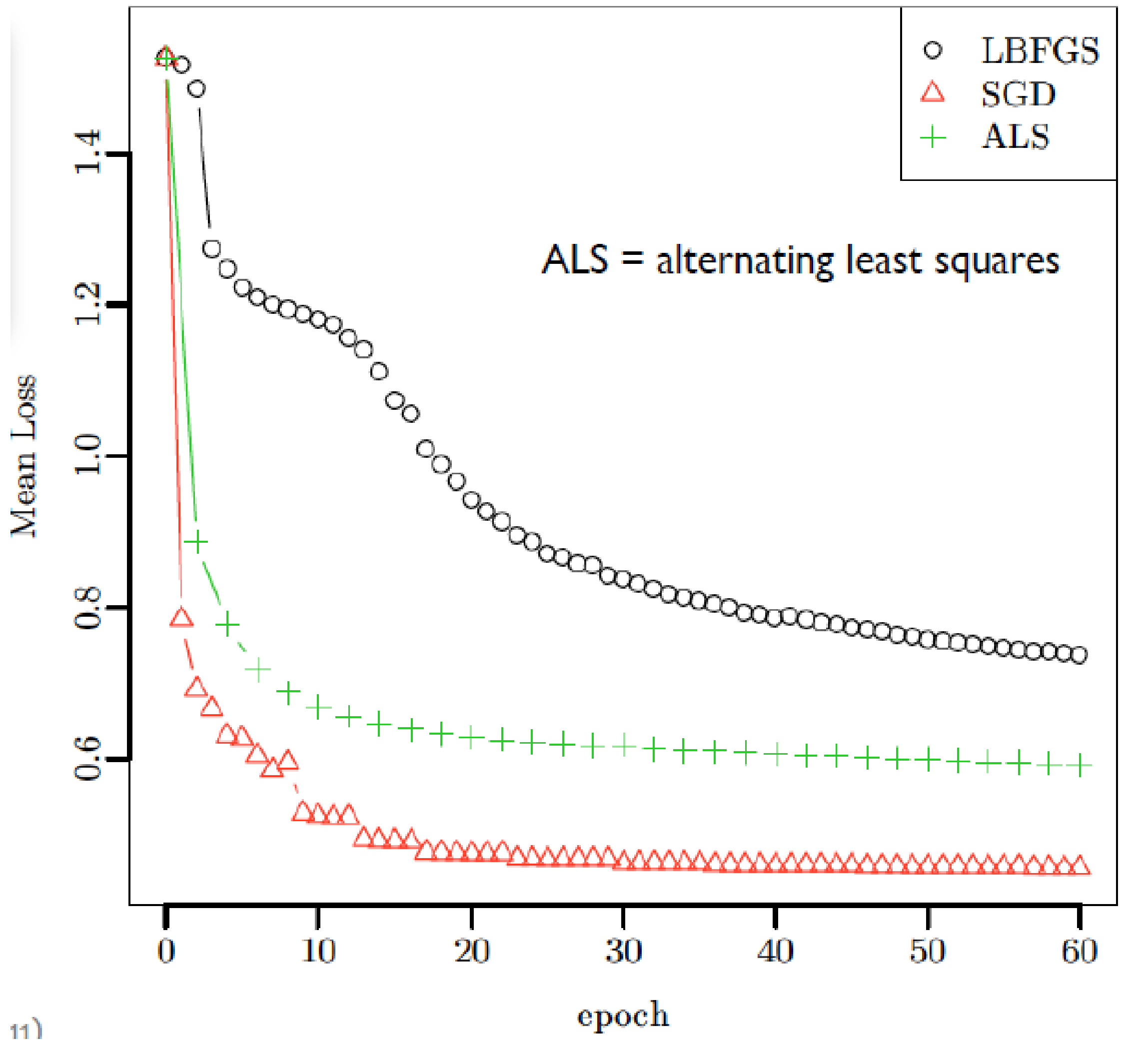


Figure 4. Matrix factorization models' accuracy. The plots show the root-mean-square error of each of four individual factor models (lower is better). Accuracy improves when the factor model's dimensionality (denoted by numbers on the charts) increases. In addition, the more refined factor models, whose descriptions involve more distinct sets of parameters, are more accurate. For comparison, the Netflix system achieves RMSE = 0.9514 on the same dataset, while the grand prize's required accuracy is RMSE = 0.8563.

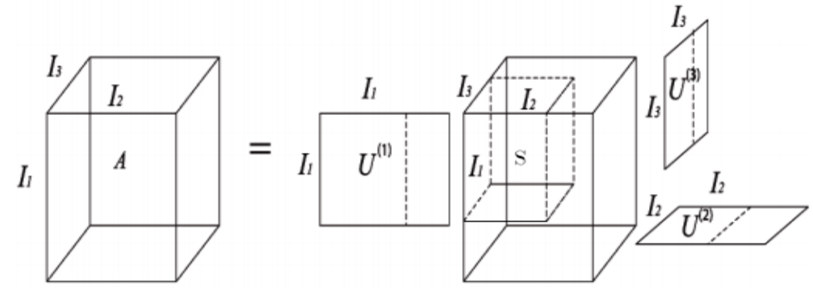
* Figures in [3]

Comparison of Optimization



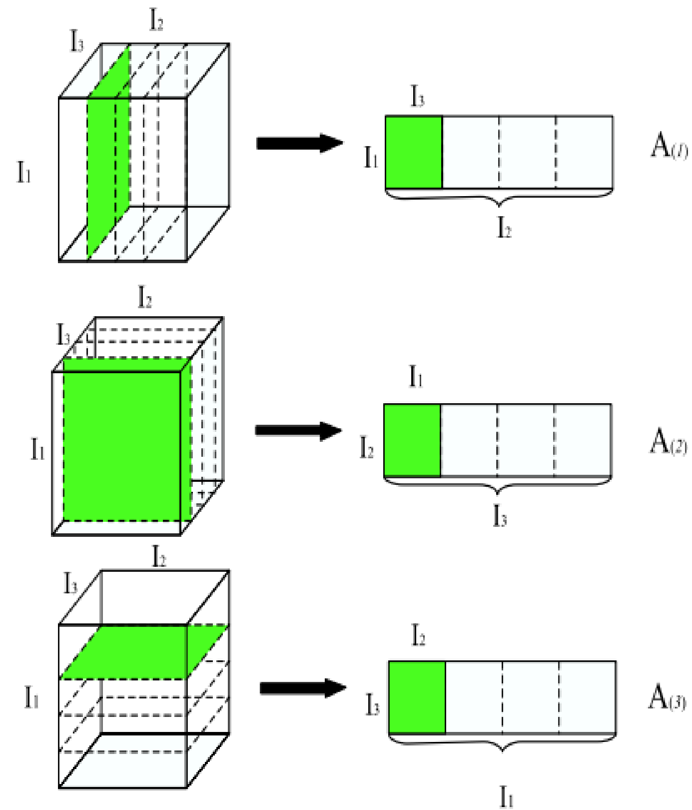
* Figures in [1]

HOSVD [4]



- SVD on each matrix

$$A_1 = U^{(1)} \cdot S_1 \cdot V_1^T$$
$$A_2 = U^{(2)} \cdot S_2 \cdot V_2^T$$
$$A_3 = U^{(3)} \cdot S_3 \cdot V_3^T$$



- Construction of core tensor

$$\mathcal{S} = \mathcal{A} \times_1 {U_{c_1}^{(1)}}^T \times_2 {U_{c_2}^{(2)}}^T \times_3 {U_{c_3}^{(3)}}^T$$

- Construction of tensor $\hat{\mathcal{A}}$

$$\hat{\mathcal{A}} = \mathcal{S} \times_1 U_{c_1}^{(1)} \times_2 U_{c_2}^{(2)} \times_3 U_{c_3}^{(3)}$$

Example

- MF in Python

```
#!/usr/bin/python

import numpy as np

np.set_printoptions(precision = 3) # set decimal display

# Matrix
A = np.zeros((5, 5))

A[0, 4] = 1
A[0, 1] = A[1, 4] = A[3, 4] = 2
A[0, 2] = A[1, 1] = A[2, 2] = A[2, 3] = 3
A[0, 0] = A[1, 3] = A[2, 1] = 4
A[0, 3] = A[2, 0] = A[3, 2] = A[3, 3] = A[4, 0] = A[4, 3] = 5

# SVD
U, s, V = np.linalg.svd(A, full_matrices = True)

# Reconstruction
S = np.diag(s)

P = np.dot(U, np.dot(S, V))
```

```
[ 4.  2.  3.  5.  1.]
[ 0.  3.  0.  4.  2.]
[ 5.  4.  3.  3.  0.]
[ 0.  0.  5.  5.  2.]
[ 5.  0.  0.  5.  0.]
```

```
[   4e+00    2e+00    3e+00    5e+00    1e+00]
[  -4e-16    3e+00    3e-15    4e+00    2e+00]
[   5e+00    4e+00    3e+00    3e+00    7e-16]
[  -4e-15    4e-15    5e+00    5e+00    2e+00]
[   5e+00    1e-15   -2e-16    5e+00   -5e-16]
```

Example

- MF in Python with r

```
#!/usr/bin/python

import rpy2.robjects as robjects

r = robjects.r

r('''
    rsvd <- function() {
        # MATRIX
        A <- matrix(c(4, 2, 3, 5, 1, 0, 3, 0, 4, 2, 5, 4, 3, 3, 0, 0, 0, 5, 5, 2, 5, 0, 0, 5, 0), nrow = 5, ncol = 5, byrow = TRUE)

        # SVD
        result <- svd(A)

        # RECONSTRUCTION
        U <- result$u
        s <- result$d
        V <- result$v

        ApproxA <- U %*% diag(s) %*% t(V)
    }
    ''')

svd = r['rsvd']

result = svd()

print result

```

	[,1]	[,2]	[,3]	[,4]	[,5]
[1,]	4	2	3	5	1
[2,]	0	3	0	4	2
[3,]	5	4	3	3	0
[4,]	0	0	5	5	2
[5,]	5	0	0	5	0

	[,1]	[,2]	[,3]	[,4]	[,5]
[1,]	4.000000e+00	2.000000e+00	3.000000e+00	5	1.000000e+00
[2,]	-1.955901e-16	3.000000e+00	5.999975e-16	4	2.000000e+00
[3,]	5.000000e+00	4.000000e+00	3.000000e+00	3	-1.110223e-16
[4,]	-6.366435e-16	3.387048e-15	5.000000e+00	5	2.000000e+00
[5,]	5.000000e+00	1.949829e-15	-6.570265e-16	5	1.408595e-15

Reference

1. Slides in "Matrix Factorization and Collaborative Filtering"
   – By Matt Gormley (Carnegie Mellon Univ.)

2. Slides in "Recommender Systems"
   – By Jee-Hyong Lee (Sungkyunkwan Univ.)

3. Y. Koren *et al*., "Matrix Factorization Techniques for Recommender Systems," Journal Computer, 42(8), 2009

4. P. Symeonidis *et.al*, "Tag Recommendations based on Tensor Dimensionality Reduction," Recsys'08